Conservaion of energy

To physicists, conservation of energy is presented as a principle of nature (Feynman might be a typical representative of this position).

However, mathematically it is only a tautology. For example, let f be a smooth real-valued function (with domain the real line). For a point t in the real line (interpreted as a point of time), we write $x = f(t)$, and also $v = f'(t), a = f''(t)$.

The expression adx then represents a differential one-form on $\mathbb{R}$, and we know it must be closed since $\mathbb{R}$ is only one-dimensional, and it must be exact then, because $\mathbb{R}$ is topologically trivial.

Then abstractly we know that there is a smooth function $g(t)$ so that, continuing our convention that we label function values by a single letter, if we write $K = g(t)$ we have

$$dK = adx.$$

Also that K is uniquely determined for all values of t once we choose an initial value.

Now, what is K? We write

$$adx = \frac{dv}{dt}dx = \frac{dx}{dt}dv = vdv = d(\frac{1}{2}v^2).$$

Hence the posible choices of K are $\frac{1}{2}v^2$ plus any constant.

This is called kinetic energy, and the fact that $adx = d(\frac{1}{2}v^2)$ means that the difference

$$\int adx - \frac{1}{2}v^2$$

is constant. By introducing a negative sign in a in some surreptitious way (every action has an equal and opposite reaction and other woffles), we can present this as if the *sum* of two things is constant, and call the other thing 'potential energy.' Thus the supposed principle that there is something called 'total energy' which is invariant.