The subsheaf generated by $F^i\mathcal{O}_V$
in degree $\frac{(r+1)^i-1}{r}$ is not finite type.

Let us consider what should be a typical example (A_2 was atypical) the case of A_3.

We represent the coordinate ring as the subalgebra $\mathcal{O}$ of $\mathbb{C}[x,y]$ generated by x^4, y^4, xy, and the The Grauert-Riemennschnieder sheaf is nothing but the graded ring

$$\oplus_{i=0}^{\infty}\mathcal{O}\omega^i$$

for $\omega = dx \wedge dy$.

It has a basis consisting of $x^i y^j \omega^s$ for which $i \equiv j\ mod 4$.

We consider now not the subalgebra generated by the L_i, but the subalgebra generated by the iterated

$$F^t(\mathcal{O}_V) \subset \mathcal{O}\omega^{\frac{(r+1)^t-1}{r}}$$

where as usual F is $r+1$'st exterior power of principal parts mod torsion. Strictly speaking writing $\omega = dx \wedge dy$ we are talking about $F(I)$ for I an ideal, as the submodule of $\mathcal{O}\omega$ generated by the

$$(f dg \wedge dh - g df \wedge dh + h df \wedge df)$$

where $f, g, h \in I$. Our result is that it is not finite type.

Lemma. Any k algebra generating set of the integral closure of the subalgebra generated by $\mathcal{O}_V$ in degree 0 and the $F^t(\mathcal{O}_V)$ in degree $\frac{(r+1)^t-1}{r}$ contains the infinite set of monomials $(xy)^{2i+1}\omega^{3i} = (xy)^{3i+1}[(xy)^{-1}\omega]^{2i}$

Corollary. The subalgebra generated by the $F^t(\mathcal{O}_V)$ is itself not finite type.

Proof of lemma. Let us just calculate some monomials. Instead of taking the exterior power of the differentials, we can multiply three affinely independent differentials and multiply by $(xy)^{-1}\omega$. $\mathcal{O}$ of course contains $1, x^4, xy, y^4$. Then $F(\mathcal{O})$ contains $1 \cdot x^4 \cdot xy \cdot (xy)^{-1}\omega = x^4\omega$ and $1 \cdot x^4 \cdot y^4 \cdot (xy)^{-1}\omega = (xy)^3\omega$ (here we note the exponent ratio $3/1$) and $1 \cdot xy \cdot xy^4 \cdot (xy)^{-1}\omega = y^4 \cdot \omega$

Now $F^2\mathcal{O}$ contains the product of these three times $(xy)^{-1}\omega$ which is $(xy)^6\omega^4$. Here we note the exponent ratio $6/4$. And it contains again $x^4\omega \cdot x^8\omega \cdot x^5y\omega \cdot (xy)^{-1}\omega = x^{16}\omega^4$ and likewise $y^{16}\omega^4$. And it also now contains $x^4 \cdot \omega \cdot x^5y \cdot \omega \cdot y^4 \cdot \omega \cdot (xy)^{-1}\omega = x^8y^4 \cdot \omega^4$ and likewise $x^4y^6 \cdot \omega^4$.

Continuing in this way we also get $x^{16}\omega^4 \cdot x^{20}\omega^4 \cdot x^{17}y\omega^4 \cdot (xy)^{-1} \cdot \omega = x^{52}\omega^{13}$ and likewise $y^{52}\omega^{13}$ and also $x^8y^4\omega^4 \cdot x^6y^6\omega^4 \cdot x^5y^9\omega^4 \cdot (xy)^{-1}\omega = (xy)^{18}\omega^{13}$ and note the relevant ratio 18/13.

So we always get powers of ω going $1, 4, 13, ...$ according to the partial sums of a geometric series to the base of 3, and we get powers of x and y multiplying these, going by $4, 16, 52, ...$ which is just four times the same sequence, and finally powers of (xy) going $3, 6, 18, 54, ...$ and going up by powers of three. So the ratio between the power of (xy) and the power of ω is $6 \cdot 3^i/(1 + ... + 3^{i+1}) = 2 \cdot 3^{i+1}/(3^{i+2} - 1)$ which tends to $2/3$. Thus the monomials $(xy)^i\omega^j$ that are in the integral closure are those for which $i/j > 2/3$. Thus the $(xy)^{2i+1}\omega^{3i}$ are in the integral closure, and it is not at all difficult to see that must be contained in any generating set.